

112 學年度資訊管理系博士班資格考

科目 Subject: Algorithms

考試方式 Type: Close book

考試時間 Time: 4 Hours

第一部份：(以下 6 題中請自行選擇 5 題作答；作答超過 5 題，僅計作答的前 5 題之分數)

Part 1: (Please choose 5 of the following 6 questions to answer; if you answer more than 5 questions, only the first 5 answered questions will be graded.)

1. What is the time complexity of a binary search tree on average and on the worst cases, respectively? Please give an example of a worst case. (10%)
2. Analyze time complexity of the following algorithm. You must justify your answer. (10%)

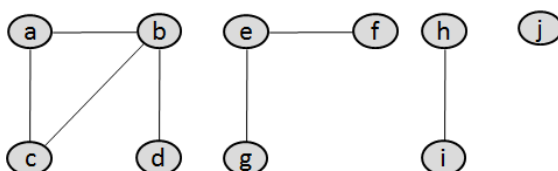
// Are all the entries in an array of int values unique from start to end?

```
public static boolean isUnique(int[] arr, int start, int end) {
    if (start >= end) return true; // the range is too small for repeats
    // check if first part of arr is recursively unique
    if (!isUnique(arr, start, end - 1)) return false;
    // check if second part of arr is recursively unique
    if (!isUnique(arr, start + 1, end)) return false;
    return (arr[start] != arr[end]); // check if first and last are different
}
```

3. Please show the result after we insert the keys 5, 28, 19, 15, 37, 33, 11, 17, 10 into a hash table with collisions resolved by **linear probing**. Let the hash function be $h(k) = k \bmod 9$. (10%)
4. Please use greedy algorithm to select a maximum-size subset of mutually compatible activities that require exclusive use of a common resource. Each activity a_i has a start time s_i and a finish time f_i . Activities a_i and a_j are compatible if the intervals $[s_i, f_i)$ and $[s_j, f_j)$ do not overlap (i.e., a_i and a_j are compatible if $s_i \geq f_j$ or $s_j \geq f_i$). (10%)

i	3	1	2	5	4	7	8	10	6	9	11
s_i	0	1	3	4	5	6	7	8	9	10	12
f_i	6	3	5	9	8	12	13	15	10	14	16

5. Given a graph $G = (V, E)$, $V = \{a, b, c, d, e, f, g, h, i, j\}$, $E = \{(a, b) (a, c) (b, c) (b, d) (e, f) (e, g) (h, i)\}$ as below, please use disjoint-set technique to find connected components in G . Show your process step by step. (10%)



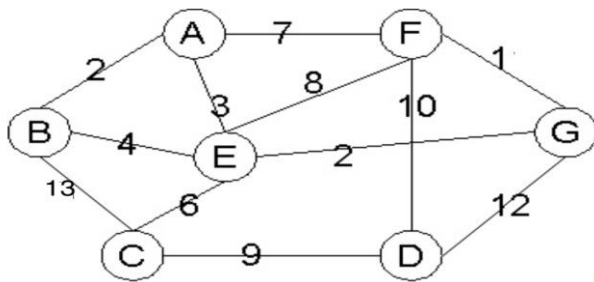
6. For a matrix chain multiplication problem, please use **dynamic programming technique** to find the **minimal number of scalar multiplications** needed to compute $A_1 A_2 A_3 A_4$. (10%)

Matrix	A_1	A_2	A_3	A_4
Dimension	35×15	15×5	5×10	10×20

第二部份：(以下 5 題請全部作答)

Part 2: (Please answer the following 5 questions.)

7. There is a binary tree with the preorder traversal sequence A B D G K H L M C E and the inorder traversal sequence K G D L H M B A E C. Please draw this binary tree. (10%)
8. Assume the Merge Sort algorithm takes 3 seconds to sort 64 pieces of data. How many data entries require 6 minutes to sort? (10%)
9. Please use Dijkstra's algorithm to find the shortest paths from point A to all other nodes in the following diagram. (10%)



10. Please compare 2-4 tree and red-black tree. (10%)
11. Please compare Breadth-First Search (BFS) and Depth-First Search (DFS) in terms of exploration strategy, data structure, memory usage, and use cases. (10%)