國立雲林科技大學資訊管理系
111 學年度第 1 學期博士班資格考
科目：演算法
時間：4 小時 (Closed book)
作答時，請注意各題之比例配分，並清楚標示題號

第一部分：50 分(選答題，5 選 4，每題 12.5 分)

1. Show how to implement a queue using two stacks. Analyze the running time of the queue operations.
2. Suppose that we have numbers between 1 and 1000 in a binary search tree, and we want to search for the number 363. Which of the following sequences could not be the sequence of nodes examined? (註：需簡要說明，僅寫 a~e 不給分)
   a. 2, 252, 401, 398, 330, 344, 397, 363.
   b. 924, 220, 911, 244, 898, 258, 362, 363.
   c. 925, 202, 911, 240, 912, 245, 363.
   d. 2, 399, 387, 219, 266, 382, 381, 278, 363.
   e. 935, 278, 347, 621, 299, 392, 358, 363
3. Demonstrate what happens when we insert the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 6 slots, and let the hash function be h(k) = k mod 6.
4. Show how quicksort can be made to run in O(n log n) time in the worst case, assuming that all elements are distinct.
5. What is the recursion tree for the computation of RECURSIVE-MATRIX-CHAIN(p,1, 4)?

```
RECURSIVE-MATRIX-CHAIN(p, i, j)
1  if i == j
2      return 0
3  m[i, j] = ∞
4  for k = i to j − 1
5      q = RECURSIVE-MATRIX-CHAIN(p, i, k)
           + RECURSIVE-MATRIX-CHAIN(p, k + 1, j)
           + p_{i−1} p_k p_j
6      if q < m[i, j]
7          m[i, j] = q
8  return m[i, j]
```

第二部分：50 分(選答題，5 選 4，每題 12.5 分)

6. Identify the following problems in class P or NP:

   (a) a problem with complexity $n^{1.5}$
   (b) a problem with complexity 2^n
   (c) a problem with complexity $\mathrm{n}log_2 n$
   (d) a problem with complexity $2^n + n^2$
   (e) a problem with complexity n!

7. Manually execute the following C program. What is the result returned by mystery(10) in the main program?

```
int mysteryhelper(int n, int s)
{
   if (n==0) return s;
   else {
        return (mysteryhelper(n-1, s)+mysteryhelper(n-1,s));
   }
}

int mystery(int n)
{
   return mysteryhelper(n, 10);
}
```

8. Given a sparse matrix M:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 7 & 0 & 0 & 0 & 8 \\ 0 & 6 & 0 & 0 & 0 \end{bmatrix}$$

(a) Show its memory representation using
   (1) row-and-column indexing.
   (2) a linked structure.

(b) Compare the advantages and disadvantages of these two schemes.

9. Given an array A=[10, 9, 8, 7, 6, 5, 4, 3, 2, 1]. Build a min heap from A using the following two approaches.

(a) The bottom-up approach.

(b) The top-down approach.

10. Compute the shortest paths from vertex A to all remaining vertices in the diagraph: