國立雲林科技大學資訊管理系
108 學年度第 1 學期博士班資格考
科目：演算法
時間：4 小時 (Closed book)
作答時，請注意各題之比例配分，並清楚標示題號

第一部分：50 分(6 題自行選 5 題作答，每題 10 分)

1. Analyze time complexity of the following algorithm. You must justify your answer. (10%)

```
// Are all the entries in an array of int values unique from start to end?
public static boolean isUnique(int[] arr, int start, int end )    {
    if (start >= end) return true;          // the range is too small for repeats
    // check if first part of arr is recursively unique
    if (!isUnique(arr, start, end－1)) return false;
    // check if second part of arr is recursively unique
    if (!isUnique(arr, start＋1, end)) return false;
    return (arr[start] !＝  arr[end]);     // check if first and last are different
}
```

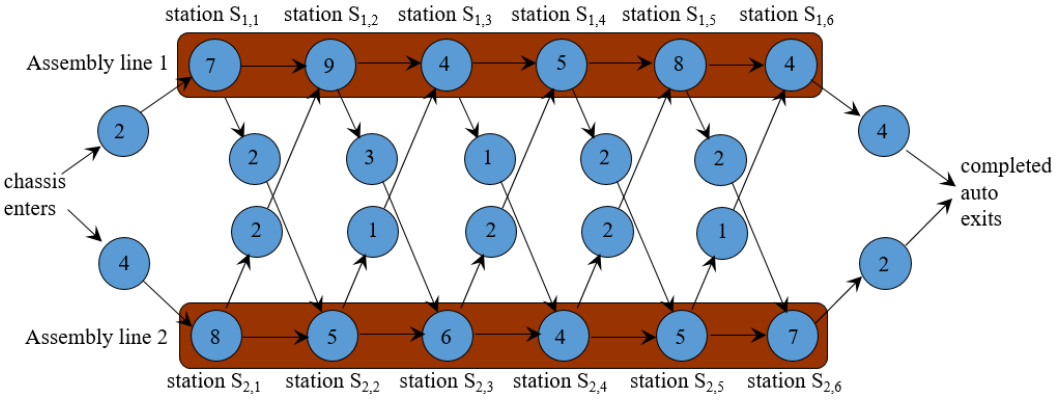
2. Please use greedy algorithm to select a maximum-size subset of mutually compatible activities that require exclusive use of a common resource. Each activity a_i has a start time s_i and a finish time f_i. Activities a_i and a_j are compatible if the intervals $[s_i, f_i)$ and $[s_j, f_j)$ do not overlap (i.e., a_i and a_j are compatible if $s_i \geq f_j$ or $s_j \geq f_i$). (10%)

i	3	1	10	2	5	4	6	7	8	9	11
s_i	0	1	2	3	3	5	5	6	8	8	12
f_i	6	4	13	5	8	7	9	10	11	12	14

3. (a) A 10,000-character data file with only six different characters. The frequency of the characters are as follows: a: 1300, b: 2100, c: 700, d: 900, e: 3500, f: 1500. Please use Huffman codes for the characters to compress the file. (5%)

   (b) How many bits are required for the data file in (a) using the Huffman codes. (5%)

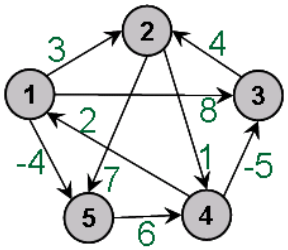
4. Using dynamic programming technique, please find the fastest path to go through all the stations and the total cost of the path. (10%)



5. Floy-Warshall algorithm computes the shortest-path weights between vertex *i* and *j* bottom up as the following formula where *k* denotes vertex number.

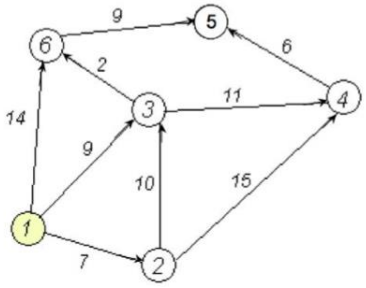
$$d_{ij}^{(k)} = \begin{cases} w_{ij} & \text{if } k = 0, \\ \min\left(d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right) & \text{if } k \geq 1. \end{cases}$$

Given the graph and $D^{(1)}$ as follows. Please compute the next step $D^{(2)}$. (10%)



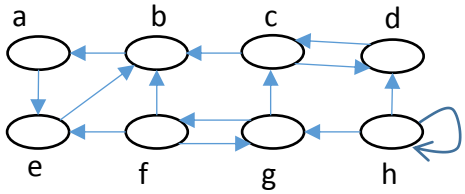
$$D^{(1)} = \begin{pmatrix} 0 & 3 & 8 & \infty & -4 \\ \infty & 0 & \infty & 1 & 7 \\ \infty & 4 & 0 & \infty & \infty \\ 2 & 5 & -5 & 0 & -2 \\ \infty & \infty & \infty & 6 & 0 \end{pmatrix}$$

6. Use the Dijkstra's Algorithm to find shortest paths from node 1 to all other nodes. (10%)

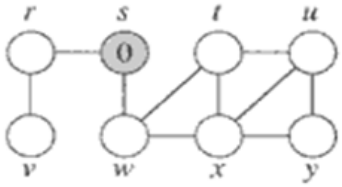


第二部分：50 分(6 題自行選 5 題作答，每題 10 分)

1. Please explain each of the following schemes for collision handling in Hashing.
   (a) Linear probing (b) Double hashing (c) Quadratic probing (d) Chaining. (10%)
2. Find strongly connected components of the graph in the following graph. (10%)

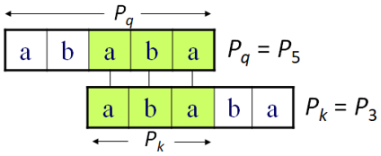


3. Please visit the following graph by using **breadth-first search** starting from node *s*. Please indicate the distance from *s* to each node in the nodes. (10%)

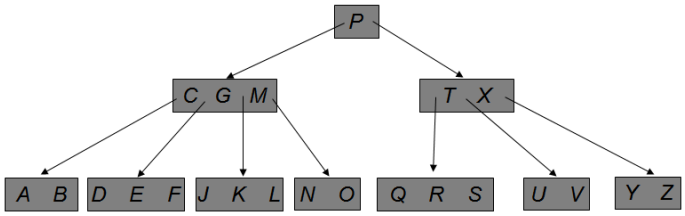


4. Given a pattern P, $\pi[q]$ is the length of the longest prefix of P that is a **proper** suffix of P_q, i.e., $\pi[q] = \max\{k : k < q \text{ and } P_k \supset P_q\}$. What are the values for $\pi[6]$~ $\pi[10]$ in the table? (10%)

i	1	2	3	4	5	6	7	8	9	10
$P[i]$	a	b	a	b	a	b	a	b	c	a
$\pi[i]$	0	0	1	2	3					



5. Assume the minimum degree $t = 3$. Please show the process of deleting the keys M and U in the given order for a B-tree as follows. (10%)



6. Determine an LCS (Longest Common Subsequence) of X = <1,0,0,1,0,1,0,1> and Y = <0,1,0,1,1,0,1,1,0> by using the dynamic programming technique. The optimal substructure of the LCS problem gives the recursive formula, where $c[i, j]$ is the length of an LCS of the sequence X_i and Y_j. (10%)

$$c[i,j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j \\ \max(c[i, j-1], c[i-1, j] & \text{if } i, j > 0 \text{ and } x_i \neq y_j \end{cases}$$