國立雲林科技大學資訊管理系
107 學年度第 1 學期博士班資格考
科目：演算法
時間：4 小時 (Closed book)
作答時，請注意各題之比例配分，並清楚標示題號

第一部分：50 分(5 題自行選 4 題作答，每題 12.5 分)

1.

Professor Diogenes has n supposedly identical integrated-circuit chips that in principle are capable of testing each other. The professor's test jig accommodates two chips at a time. When the jig is loaded, each chip tests the other and reports whether it is good or bad. A good chip always reports accurately whether the other chip is good or bad, but the professor cannot trust the answer of a bad chip. Thus, the four possible outcomes of a test are as follows:

Chip A says	Chip B says	Conclusion
B is good	A is good	both are good, or both are bad
B is good	A is bad	at least one is bad
B is bad	A is good	at least one is bad
B is bad	A is bad	at least one is bad

Show that if more than $n/2$ chips are bad, the professor cannot necessarily determine which chips are good using any strategy based on this kind of pairwise test. Assume that the bad chips can conspire to fool the professor.

2. What is the recursion tree for the computation of RECURSIVE-MATRIX-CHAIN(p,1, 4)?

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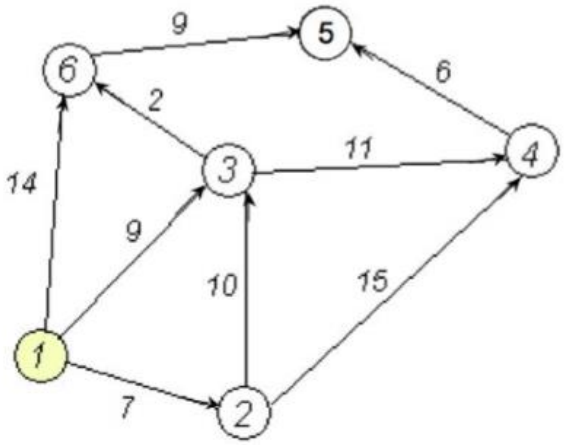
RECURSIVE-MATRIX-CHAIN(p, i, j)
1  if i == j
2      return 0
3  m[i, j] = ∞
4  for k = i to j − 1
5      q = RECURSIVE-MATRIX-CHAIN(p, i, k)
           + RECURSIVE-MATRIX-CHAIN(p, k + 1, j)
           + p_{i−1} p_k p_j
6      if q < m[i, j]
7          m[i, j] = q
8  return m[i, j]
```

3.

Explain why the worst-case running time for bucket sort is $\Theta(n^2)$. What simple change to the algorithm preserves its linear average-case running time and makes its worst-case running time $O(n \lg n)$?

4. Use the Dijkstra's Algorithm to find shortest paths from node 1 to all other nodes.



5. Prove that the traveling-salesman problem is NP-complete.

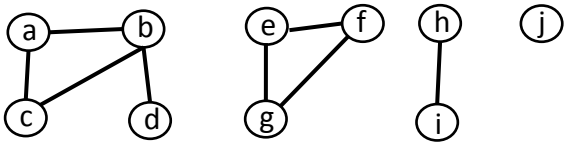
第二部分：50 分(7 題自行選 5 題作答，每題 10 分)

1. Please use Quicksort to sort the following numbers: 70, 20, 45, 62, 15, 95, 30, 55, 80, 41. You need to show each step of the sorting process.
2. Demonstrate what happens when we insert the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \bmod 9$.
3. Discuss what is meant by the O-notation: $O(g(n))$.

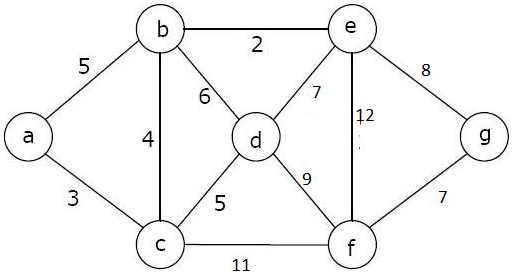
4. Determine an LCS (Longest Common Subsequence) of X = <1,0,0,1,0,1,0,1> and Y = <0,1,0,1,1,0,1,1,0> by using the dynamic programming technique. The optimal substructure of the LCS problem gives the recursive formula, where $c[i, j]$ is the length of an LCS of the sequence X_i and Y_j.

$$c[i,j] = \begin{cases} 0 & \text{if } i = 0 \text{ or } j = 0 \\ c[i-1, j-1] + 1 & \text{if } i, j > 0 \text{ and } x_i = y_j \\ \max(c[i, j-1], c[i-1, j] & \text{if } i, j > 0 \text{ and } x_i \neq y_j \end{cases}$$

5. Please apply the disjoin-set data structure to find the connected components of the following graph. You need to show each step of the process.



6. Kruskal's algorithm finds a safe edge to add to the growing forest by finding, of all the edges that connect any two trees in the forest, an edge (u,v) of least weight. Please use Kruskal's algorithm to find the minimum spanning tree of the following graph.



7. The transitive closure of a graph G is the graph G^* = (V, E^*), where

$$E^* = \{(i,j): \text{there is a path from vertex } i \text{ to vertex } j \text{ in } G\}.$$

Please find the transitive closure G^* of the following graph. Present your answer by using a two-dimensional matrix where value 1 in a cell indicates there is a path and otherwise 0.