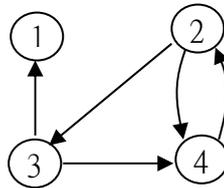


1. (a) Design an algorithm based on dynamic programming for finding the longest common subsequence (LCS) of two strings X and Y . (8%) (b) Use the algorithm to find the LCS of $X = \langle A, B, C, B, D, A, B, C, B \rangle$ and $Y = \langle B, D, C, A, B, A, C, B \rangle$. (7%).
2. What is a binomial heap? How many binomial trees does an n -node binomial heap consists of at most? (10%)
3. (a) Design an algorithm based on the Floyd-Warshall algorithm for computing the transitive closure of a graph. The Floyd-Warshall algorithm solve the all-pairs shortest-paths problem by considering the “intermediate” vertices of a shortest path, where an intermediate vertex of a simple path $p = \langle v_1, v_2, \dots, v_l \rangle$ is any vertex of p other than v_1 or v_l , that is, any vertex in the set $\{v_2, v_3, \dots, v_{l-1}\}$. (8%) (b) Use the algorithm to compute the transitive closure of the given figure. You need to show the process. (7%)



4. (a) Design a recursive quicksort algorithm. (5%) (b) Use the algorithm to sort the array $A = [2, 8, 7, 1, 3]$. You need to show your process. (5%)
5. Write a recursive procedure OS-KEY-RANK(T, k) that takes as input an order-statistic tree T and a key k and returns the rank of k in the dynamic set represented by T . Assume that the keys of T are distinct. (10%)
6. Generalize Huffman’s algorithm to ternary codewords (i.e., codewords using the symbols 0, 1, 2), and prove that it yields optimal ternary codes. (15%)
7. Suppose that the keys $\{1, 2, \dots, n\}$ are inserted into an empty B-tree with minimum degree 2. How many nodes does the final B-tree have? (15%)
8. Solve the equation

$$\begin{pmatrix} 1 & 5 & 4 \\ 2 & 0 & 3 \\ 5 & 8 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 9 \\ 5 \end{pmatrix}$$

By using an LUP decomposition. (10%)