95 Ph.D. Qualify Exam., Subject: Algorithms

Four hours, Closed book.

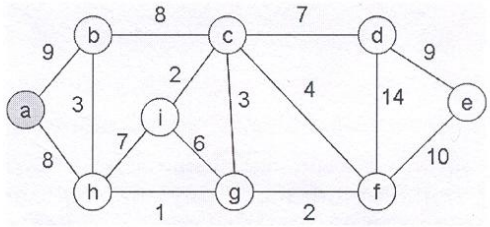
1. The matrix-chain multiplication problem can be stated as follows: given a chain $\langle A_1, A_2, \ldots, A_n \rangle$ of n matrices, where for $i = 1, 2, \ldots, n$, matrix A_i has dimension $p_{i-1} \times p_i$, fully parenthesize the product $A_1A_2 \ldots A_n$ in a way that minimizes the number of scalar multipliaitons. Please describe an approach based on dynamic programming for solving the matrix-chain multiplication problem. (15%)

2. A disjoint-set data structure maintains a collection $S = \{S_1, S_2, \ldots, S_k\}$ of disjoint dynamic sets. A simple way to implement a disjoint-set data structure is to represent each set by a linked list.

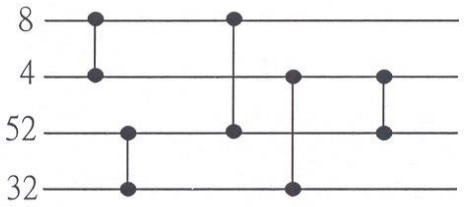
   (a) Given two sets S_1={c, h, e, b} and S_2={f, g, d}, please use figures to show how link lists represent these two sets. (5%)

   (b) Moreover, please design an algorithm UNION(x, y) to unite the dynamic sets that contain x and y, say S_x and S_y, into a new set that is the union of these two sets, and an algorithm FIND-SET(x) to return a pointer to the representative of the (unique) set containing x. (10%)

3. Given the following graph, to start from node a, find the minimum spanning tree of the graph using the Prim's algorithm. You don't need to write the algorithm. However, you need to show your process of solving the problem by drawing a figure for each step. Hint: Prim's algorithm uses a min-priority queue Q to help select next edge for processing. (10%)



4. Given a 4-input, 4-output comparison network, which is in fact a sorting network, as the following. What is the output of the network? How many time steps does it take to generate the output? (10%)



5. Show the results of inserting the keys (10%)

   F, S, Q, K, C, L, H, T, V, W, M, R, N, P, A, B, X, Y, D, Z, E

   in order into an empy B-tree with minimum degree 2. Only draw the configurations of the tree just before some node must split, and also draw the final configuration of each insertion step.

6. Give a linear-time algorithm to determine if a text T is a cyclic rotation of another string T'. For example, ***arc*** and ***car*** are cyclic rotations of each other. (15%)

7. Explain what is the circuit satisfiablity problem (5%).

8. Which of the folloing statements are true ? why, why not? (10%)

   (a) The lower bound of NP-complete problem is exponential if and only if NP $\neq$P is proved

   (b) We have not been able to prove that efficient algorithms cannot exist for solving any NP-complete problem

9. Draw a state-transition diagram for a string-matching automaton for the pattern ababbabbababbababbabb over the alphabet Σ= {a, b} (10%).