

Ph.D. Qualify Exam., Subject: Algorithms

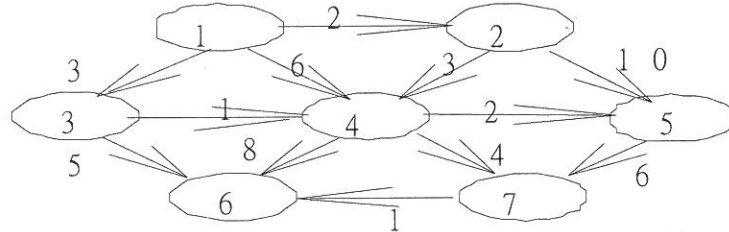
Four hours, Closed book. (103.09.02)

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1. Show that the solution of $T(n) = T(\lfloor n/2 \rfloor) + 1$ is $O(\lg n)$. (10%)
2. Suppose that n balls are tossed into n bins, where each toss is independent and the ball is equally likely to end up in any bin. What is the expected number of empty bins? What is the expected number of bins with exactly one ball?(10%)
3. Explain under what circumstances, if any, redundant DISK-READ or DISK-WRITE operations are performed during the course of executing a call to B-TREE-INSERT. (A redundant DISK-READ is a DISK-READ for a page that is already in memory. A redundant DISK-WRITE writes to disk a page of information that is identical to what is already stored there.) (10%)
4. Suppose we have an optimal prefix code on a set $C = \{0, 1, \dots, n-1\}$ of characters and we wish to transmit this code using as few bits as possible. Show how to represent any optimal prefix code on C using only $2n - 1 + n\lceil \lg n \rceil$ bits. (*Hint*: Use $2n - 1$ bits to specify the structure of the tree, as discovered by a walk of the tree.) (10%)
5. There are two types of professional wrestlers : "good guys" and "bad guys." Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$ -time algorithm that determines whether it is possible to designate some of the wrestlers as good guy and the remainder as bad guys such that each rivalry is between a good guy and a bad guy. If it is possible to perform such a designation, your algorithm should produce it. (10%)
6. 試寫出以下程式的Big-O為何? 並簡單說明為什麼? (10%)。

```
piece_of_code (int n)
{
    if (n > 1)
        piece_of_code ( n/3);
    else
        return (30000000);
}
```

7. 請以最短路徑的演算法(Dijkstra教授所提之演算法)，找出從點(1)到其餘各點的最短路徑的成本，並列出到每個節點的最短路徑 (10%)。



8. 從一個空的AVL的樹開始，依以下資料的插入順序，請依英文字母大小排列的順序，建立一個二元AVL樹：APR, MAR, JUL, AUG, DEC, JAN, OCT, FEB, NOV, MAY, JUN。在每次的資料插入後，請繪出其AVL樹，並註明在每一次資料插入後，經過旋轉後的AVL樹(如果有旋轉的情形) (10%)。
9. 請利用 **Quicksort** 的排序方式，排序右邊的值: 43, 51, 45, 41, 22, 12, 99, 53, 93, 21 (10 points)(務必寫出每個步驟的結果) (10%)。
10. 請寫出「河內塔」副程式(函數)，即完成下列hanoi副程式空白部份 (10分)

```
#include <stdio.h>
#include <stdlib.h>

int hanoi (int, char, char, char);

main ()
{
    int n,count;

    printf("請輸入盤數：");
    scanf("%d", &n);
    count=hanoi(n, 'A', 'B', 'C');
    printf("共搬移%d 次\n",count);
    system("PAUSE");

    return 0;
}

int hanoi(int n, char A, char B, char C)
{

```