

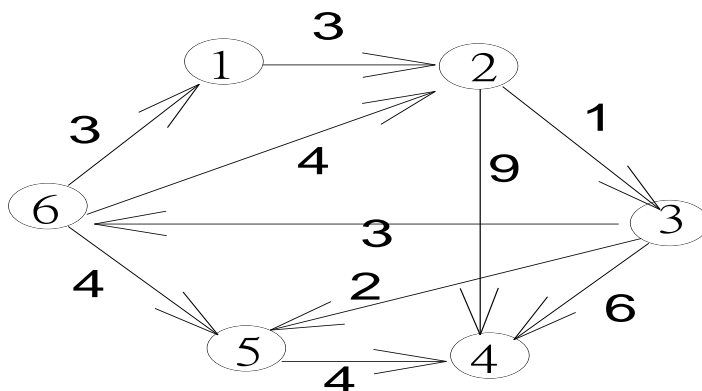
2013 Spring Qualifying Exam

- 一、What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers (A:1, B:1, C:2, D:3, E:5, F:8, G:13, H:21)? Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers? (**10 points**)
- 二、Consider the problem of making change for n cents using the fewest number of coins. Assume that each coin's value is an integer. Describe a greedy algorithm to make change consisting of quarters, dimes, nickels, and pennies. Prove that your algorithm yields an optimal solution. (**10 points**)
- 三、假設某個地區有六個城，城與城間之間的距離(成本)，請參考下表。請用 Greedy Algorithm 找出一個路線，而這個路線的規則是已知從點“1”出發，必須經過除“1”以外的每個城一次(最多也是一次)，最後回到點“1”的城。試問您所找出路線是否是最佳的路線。如果是的話，請簡略說明為什麼是最佳的路線及路線的總成本為何。如果不是的話，請簡略說明路線的總成本及為什麼不是最佳的路線，而那個路線是最佳的路線及該路線的總成本。 (**10 points**)

From	To:	2	3	4	5	6
1		6	12	5	7	25
2			9	12	18	26
3				11	14	20
4					5	15
5						8

- 四、A single server has n customers to serve. The service time required by each customer is known in advance: customer i will take time t_i , $1 \leq i \leq n$ 。如果您是那位server 的話，試問您會以什麼順序來服務這些customer，使得他們的等待時間為最短，並證明您的論點(**10 points**)。

五、請用Dijkstra的演算法(最短路徑尋找法)找出下列圖的從點“1”到其它點間的最小成本 (10 points)。



6. What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? What about in decreasing order? (10 points)

7. Proof or explain:

Inserting an element into an open-address hash table with load factor α requires at most $1/(1-\alpha)$ probes on average, assuming uniform hashing. (10 points)

8. Tell me what is “Treap”? (10 points)

9. How to augment a data structure? (10 points)

10. Proof:

If $n \geq 1$, then for any n -key B-tree T of height h and minimum degree $t \geq 2$,

$$h \leq \log_t \frac{n+1}{2}. \text{ (10 points)}$$